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Spin chains in $\mathcal{N} = 6$ superconformal Chern-Simons-Matter theory

Davide Gaiotto,[a] **Simone Giombi**[b] **and Xi Yin**[b]

[a] *Institute for Advanced Study, Princeton, NJ, U.S.A.*
[b] *Jefferson Physical Laboratory, Harvard University, Cambridge, MA 02138 U.S.A.*
E-mail: dgaiotto@gmail.com, giombi@physics.harvard.edu, xiyin@fas.harvard.edu

Abstract: In this note we study spin chain operators in the $\mathcal{N} = 6$ Chern-Simons-matter theory recently proposed by Aharony, Bergman, Jafferis and Maldacena to be dual to type IIA string theory in $AdS_4 \times \mathbb{CP}^3$. We study the two-loop dilatation operator in the gauge theory, and compare to the Penrose limit on the string theory side.

Keywords: Penrose limit and pp-wave background, AdS-CFT Correspondence, Chern-Simons Theories

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JHEP04(2009)066

Contents

1 Introduction

The long standing problem of finding an exact description of the CFT dual to M-theory on $AdS_4 \times S^7$ (and orbifolds thereof), or the low energy limit of the world volume theory of N coinciding M2-branes, was solved beautifully in a recent paper of Aharony, Bergman, Jafferis and Maldacena [1]. The dual gauge theory is a special case of the $\mathcal{N} = 3$ superconformal Chern-Simons-matter (CSM) theories studied in [10] (see [11–15] for earlier works), which has quiver type matter content and enhanced $\mathcal{N} = 6$ supersymmetry. In particular, the 't Hooft limit of the $\mathcal{N} = 6$ CSM theory is argued to be dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$. See also [9, 16–20] for subsequent works on this theory, and [2–8] for recent works on M2-brane world volume theories.

In this paper we make a step toward understanding the details of the duality between the $\mathcal{N} = 6$ CSM theory and type IIA string on $AdS_4 \times \mathbb{CP}^3$ in non(near)-BPS sectors, by exploring both spin chain operators in the superconformal gauge theory (continuing on [10]) and the Penrose limit of the string theory dual. We study the two-loop dilatation operators in subsectors of the spin chain, as well as the dispersion relation and scattering of impurities in an infinite chain that preserves a centrally extended SU(2|2) superconformal algebra. The central charge of the SU(2|2) algebra plays a key role in determining the exact dispersion relations of the impurities. It is related to the momentum P along the spin chain in the form

$$Z = f(\lambda)\left(1 - e^{2\pi i P}\right)$$

JHEP04(2009)066

where $f(\lambda)$ is a nontrivial function of the 't Hooft coupling $\lambda = N/k$. We find that $f(\lambda)$ scales differently with λ at weak coupling (from perturbative gauge theory) and at strong coupling (from the Penrose limit). We discuss operator mixing and match multiplets in the weak coupling regime with those in the pp-wave limit. We also present some preliminary discussions on the giant magnons in $AdS_4 \times \mathbb{CP}^3$.

Note added in proof: Upon completion of the bulk of this work, we received [19] and [16], which contain results that overlap with different parts of this paper.

2 The $\mathcal{N} = 6$ Chern-Simons-Matter theory

2.1 Lagrangian

It will be useful for us to formulate $\mathcal{N} = 6$ Chern-Simons-matter theory in the $\mathcal{N} = 2$ language. The gauge group will be $\mathrm{U}(N) \times \mathrm{U}(N)$, with a pair of chiral fields A_i $(i = 1, 2)$ in the bifundamental representation $(\mathbf{N}, \bar{\mathbf{N}})$, and B_i in the conjugate representation $(\bar{\mathbf{N}}, \mathbf{N})$. There is an $\mathcal{N} = 2$ superpotential

$$W = \frac{4\pi}{k}\mathrm{Tr}\,(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) \tag{2.1}$$

This theory possesses $\mathcal{N} = 6$ supersymmetry, and is exactly conformal, with superconformal group $O\mathrm{Sp}(6|4)$. The scalar components of $(A_1, A_2, B_1^\dagger, B_2^\dagger)$ transform in the $\mathbf{4}$ of $\mathrm{SU}(4)_R$, whereas $(B_1, B_2, A_1^\dagger, A_2^\dagger)$ transform in the $\bar{\mathbf{4}}$.

The scalar potential can be written as $V = V_D + V_F$, where $V_F = |\partial W/\partial A_i|^2 + |\partial W/\partial B_i|^2$, and V_D comes the coupling of the scalar fields to the auxiliary fields σ and $\tilde{\sigma}$ (which lie in the $\mathcal{N} = 2$ gauge multiplet and take values in the adjoint of the two $\mathrm{U}(N)$'s),

$$V_D = \mathrm{Tr}\left[(\sigma A_i - A_i\tilde{\sigma})\left(A_i^\dagger\sigma - \tilde{\sigma}A_i^\dagger\right)\right] + \mathrm{Tr}\left[(\tilde{\sigma}B_i - B_i\sigma)\left(B_i^\dagger\tilde{\sigma} - \sigma B_i^\dagger\right)\right] \tag{2.2}$$

where

$$\begin{aligned}\sigma &= \frac{2\pi}{k}\left(A_i A_i^\dagger - B_i^\dagger B_i\right),\\ \tilde{\sigma} &= -\frac{2\pi}{k}\left(B_i B_i^\dagger - A_i^\dagger A_i\right).\end{aligned} \tag{2.3}$$

There are quartic boson-fermion coupling of the form

$$\begin{aligned}\mathcal{L}_F = \mathcal{L}_Y &- \mathrm{Tr}\left(\psi_{A_i}^\dagger\sigma\psi_{A_i} - \psi_{A_i}^\dagger\psi_{A_i}\tilde{\sigma}\right) - \mathrm{Tr}\left(\psi_{B_i}^\dagger\tilde{\sigma}\psi_{B_i} - \psi_{B_i}^\dagger\psi_{B_i}\sigma\right)\\ &- \mathrm{Tr}\left(A_i^\dagger\chi^\dagger\psi_{A_i} - \tilde{\chi}^\dagger A_i^\dagger\psi_{A_i} + c.c.\right) - \mathrm{Tr}\left(B_i^\dagger\tilde{\chi}^\dagger\psi_{B_i} - \chi^\dagger B_i^\dagger\psi_{B_i} + c.c.\right)\end{aligned} \tag{2.4}$$

where χ and $\tilde{\chi}$ are fermionic auxiliary fields in the $\mathcal{N} = 2$ gauge multiplet,

$$\begin{aligned}\chi &= \frac{2\pi}{k}\left(\psi_{A_i}A_i^\dagger - B_i^\dagger\psi_{B_i}\right),\\ \tilde{\chi} &= -\frac{2\pi}{k}\left(\psi_{B_i}B_i^\dagger - A_i^\dagger\psi_{A_i}\right).\end{aligned} \tag{2.5}$$

and $\mathcal{L}_Y$ is the Yukawa coupling,

$$\begin{aligned}\mathcal{L}_Y &= \frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + c.c.\\ &= \frac{4\pi}{k}\mathrm{Tr}\,(A_1 B_1\psi_{A_2}\psi_{B_2} + \cdots)\end{aligned} \tag{2.6}$$

2.2 Supersymmetry transformations

In manifestly $\mathcal{N}=6$ supersymmetric notation, we can write the supercharges as $Q_{IJ} = (Q^{IJ})^\dagger = \frac{1}{2}\epsilon_{IJKL}\bar{Q}^{KL}$, where $I,J,K,L = 1,\cdots,4$. The scalars and fermions are denoted by ϕ_I, $\bar{\phi}^I$, $(\psi_I)_\alpha$ and $\bar{\psi}^I_\alpha$. One can explicitly identify them with the components fields of $\mathcal{N}=2$ chiral multiplets as

$$\begin{aligned} &\phi_1 = A_1, \qquad \phi_2 = A_2, \qquad \phi_3 = B_1^\dagger, \qquad \phi_4 = B_2^\dagger, \\ &\psi_1 = -\psi^\dagger_{A_2}, \qquad \psi_2 = \psi^\dagger_{A_1}, \qquad \psi_3 = -\psi_{B_2}, \qquad \psi_4 = \psi_{B_1}. \end{aligned} \tag{2.7}$$

The action of the supercharges on the fields is as follows

$$\begin{aligned} Q_{IJ}\phi_K &= \epsilon_{IJKL}\bar{\psi}^L, \\ Q_{IJ}\bar{\phi}^K &= \delta^K_I\psi_J - \delta^K_J\psi_I, \\ (Q_{IJ})_\alpha(\psi_K)_\beta &= \epsilon_{IJKL}i\sigma^\mu_{\alpha\beta}D_\mu\bar{\phi}^L + \frac{2\pi i}{k}\epsilon_{\alpha\beta}\epsilon_{IJKL}\left(\bar{\phi}^L\phi_M\bar{\phi}^M - \bar{\phi}^M\phi_M\bar{\phi}^L\right) \\ &\quad + \frac{4\pi i}{k}\epsilon_{\alpha\beta}\epsilon_{IJLM}\bar{\phi}^L\phi_K\bar{\phi}^M, \\ (Q_{IJ})_\alpha(\bar{\psi}^K)_\beta &= \delta^K_I\left[i\sigma^\mu_{\alpha\beta}D_\mu\phi_J - \frac{2\pi i}{k}\epsilon_{\alpha\beta}\left(\phi_J\bar{\phi}^M\phi_M - \phi_M\bar{\phi}^M\phi_J\right)\right] \\ &\quad - \delta^K_J\left[i\sigma^\mu_{\alpha\beta}D_\mu\phi_I - \frac{2\pi i}{k}\epsilon_{\alpha\beta}\left(\phi_I\bar{\phi}^M\phi_M - \phi_M\bar{\phi}^M\phi_I\right)\right] \\ &\quad - \frac{4\pi i}{k}\epsilon_{\alpha\beta}(\phi_I\bar{\phi}^K\phi_J - \phi_J\bar{\phi}^K\phi_I), \\ Q_{IJ}A_\mu &= i\sigma_\mu\chi_{IJ} = \frac{2\pi i}{k}\sigma_\mu\left(\phi_{[I}\psi_{J]} + \frac{1}{2}\epsilon_{IJKL}\bar{\psi}^K\bar{\phi}^L\right), \\ Q_{IJ}\tilde{A}_\mu &= i\sigma_\mu\tilde{\chi}_{IJ} = \frac{2\pi i}{k}\sigma_\mu\left(\psi_{[J}\phi_{I]} + \frac{1}{2}\epsilon_{IJKL}\bar{\phi}^L\bar{\psi}^K\right). \end{aligned} \tag{2.8}$$

3 Spin chains in $\mathcal{N}=6$ CS

3.1 $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ sector

Let us focus on the $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B \times \mathrm{U}(1)$ subgroup of $\mathrm{SU}(4)_R$, where A_i transform in the representation $(\mathbf{2},\mathbf{1},+1)$, and B_i in the representation $(\mathbf{1},\mathbf{2},-1)$. Consider spin chains of the form

$$\mathrm{Tr}\,(A_{i_1}B_1A_{i_2}B_1A_{i_3}B_1\cdots) \tag{3.1}$$

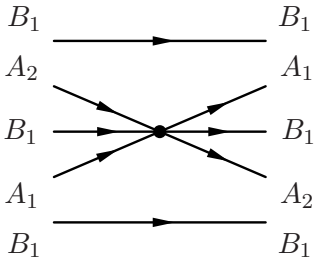
These are chiral operators, but in general not primaries due to the superpotential. At two-loop, the sextic scalar potential coming from the superpotential contributes to the anomalous dimension of the above operator. The relevant potential term is

$$\frac{16\pi^2}{k^2}\mathrm{Tr}\left[(A_1B_1A_2 - A_2B_1A_1)\,(A_1B_1A_2 - A_2B_1A_1)^\dagger\right] \tag{3.2}$$

The potential terms in V_D does not contribute at two-loop. Similarly, the terms coupling the scalars to fermions in $\mathcal{L}_F$ do not have the right structure to contribute to the two-loop

JHEP04(2009)066

anomalous dimension of the chiral operator either (other than an overall shift which is fixed by the BPS bound for the chiral primaries, i.e. the operators with all the A_i's symmetrized).



The two-loop integral in the above diagram is

$$\int \frac{d^3y}{(4\pi)^6} \frac{1}{|y|^3|x-y|^3} \sim \frac{1}{8\pi^2} \frac{\ln \Lambda}{(4\pi|x|)^3} \tag{3.3}$$

where $1/(4\pi|x|)$ is the scalar propagator in position space. There is also a factor of $16\pi^2\lambda^2$ from the vertices and contraction of color indices, and a factor of $1/2$ since we were calculating the two point function of the spin chain operator as opposed to the anomalous dimension. Putting these together, we then find the two-loop spin chain Hamiltonian

$$H = -\lambda^2 \sum_i (P_{i,i+1} - 1) \tag{3.4}$$

This is the Hamiltonian of the Heisenberg XXX spin-1/2 chain. The dispersion relation of an impurity in this SU(2) sector moving with momentum p is

$$E = 4\lambda^2 \sin^2(\pi p) + \mathcal{O}(\lambda^3) \tag{3.5}$$

There may be a regularization scheme dependent order λ^3 term, but its structure is the same as the λ^2, since the corresponding three loop diagrams are obtained by attaching gauge propagators to the two-loop diagrams.

Now let us allow the B_1's to change into B_2 as well, so that the spin chain takes the form

$$\mathrm{Tr}\,(A_{i_1} B_{j_1} A_{i_2} B_{j_2} A_{i_3} B_{j_3} \cdots) \tag{3.6}$$

Once again, at two-loop the only potential term that contributes to the anomalous dimension are the ($\mathcal{N} = 2$) F-terms. Furthermore, the exchanges of A_1 and A_2 across B_1 or B_2 have the same amplitude, and similarly for the exchange of B_1 and B_2 across A_i. Therefore, we find that at two-loop the $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ spin chain is two decoupled XXX spin-1/2 chains (of A's and B's respectively).

3.2 The SU(2|2) infinite chain

To gain further insight we shall consider the infinite chain (the "vacuum")

$$\mathrm{Tr}\,(A_1 B_1 A_1 B_1 A_1 B_1 \cdots) \tag{3.7}$$

It preserves an SU(2|2) subgroup of $O\mathrm{Sp}(6|4)$. The bosonic part of SU(2|2) is $\mathrm{SU}(2)_G \times \mathrm{SU}(2)_r \times \mathrm{U}(1)_D$, where $\mathrm{SU}(2)_G$ rotates $A_2, B_2^\dagger$ as a doublet, $\mathrm{SU}(2)_r$ is the rotation group

JHEP04(2009)066

in spacetime, and $\mathrm{U}(1)_D$ is generated by D, defined to be the anomalous dimension. More precisely, $D = \Delta - J$, where Δ is the conformal dimension and J is the eigenvalue of the Cartan generator of $\mathrm{SU}(2)_{G'}$, which is the group rotating $A_1, B_1^\dagger$ (and similarly ψ_1, ψ_3) as a doublet.[1] Therefore one has $J(A_1) = J(B_1) = \frac{1}{2}, J(A_2) = J(B_2) = 0$, and similarly for the fermions. The odd generators of SU(2|2) are denoted by $Q_{A\alpha}, \bar{S}_{A\alpha}$, where A is an $\mathrm{SU}(2)_G$ doublet index, and α is the spacetime spinor index. The superalgebra is

$$\begin{aligned} &\{Q_{A\alpha}, Q_{B\beta}\} = \epsilon_{AB}\epsilon_{\alpha\beta} Z, \quad \{\bar{S}_{A\alpha}, \bar{S}_{B\beta}\} = \epsilon_{AB}\epsilon_{\alpha\beta}\bar{Z}, \\ &\{Q_{A\alpha}, \bar{S}_{B\beta}\} = \epsilon_{AB}\epsilon_{\alpha\beta} D + \epsilon_{AB} J_{\alpha\beta} + \epsilon_{\alpha\beta} T_{AB}. \end{aligned} \tag{3.8}$$

where Z is a central charge, related to the momentum of the impurities in the infinite chain, to be determined later.

Comparing with the supersymmetry transformations (2.8), the pair of supercharges that preserve the vacuum spin chain is $(Q_{12}, -Q_{14}) \sim Q_A$. In particular, $J(Q_A) = \frac{1}{2}$, and $D = \Delta - J$ commutes with the supercharges as required by the SU(2|2) algebra.

The basic impurities are $A_2, B_2^\dagger, (\psi_{B_2}^\dagger)_\alpha$ in place of A_1, and similarly $A_2^\dagger, B_2, (\psi_{A_2}^\dagger)_\alpha$ in place of B_1. At zero momentum they transform in the minimal short representation of SU(2|2). We will write $\phi_A = (A_2, B_2^\dagger) = (\phi_2, \phi_4)$, and $\chi_\alpha = (\psi_{B_2}^\dagger)_\alpha$. From (2.8) we have the supersymmetry transformations on (ϕ_A, χ_α)

$$\begin{aligned} &Q_{A\alpha}\phi_B \sim \epsilon_{AB}\chi_\alpha, \\ &Q_{A\alpha}\chi_\beta \sim \epsilon_{\alpha\beta}\frac{2\pi i}{k}(\phi_A B_1 A_1 - A_1 B_1 \phi_A). \end{aligned} \tag{3.9}$$

In terms of impurities with momentum p, we have

$$\begin{aligned} &Q_{A\alpha}|\phi_B(p)\rangle \sim \epsilon_{AB}|\chi_\alpha(p)\rangle, \\ &Q_{A\alpha}|\chi_\beta(p)\rangle \sim \epsilon_{\alpha\beta}\frac{2\pi i}{k}(1 - e^{2\pi i p})|\phi_A(p)\rangle. \end{aligned} \tag{3.10}$$

In (3.8) we have normalized Q_A and $\bar{S}_A$ to be complex conjugates of one another in radial quantization. In general they are related to the supercharges in (2.8) by a rescaling, which a priori may depend on the coupling λ due to quantum corrections to $\bar{S}_A$. The central charge of the SU(2|2) algebra takes the form $Z = f(\lambda)(1 - e^{2\pi i p})$, where $f(\lambda)$ is an undetermined function of λ. The basic impurities (4 bosonic and 4 fermionic) fall into two short representations:

$$\{[1,0]|[0,1]\} \oplus \{[1,0]|[0,1]\} \tag{3.11}$$

We will call them $(2|2)_A$ and $(2|2)_B$ impurities for short. The short multiplet saturates the BPS bound [22, 23],

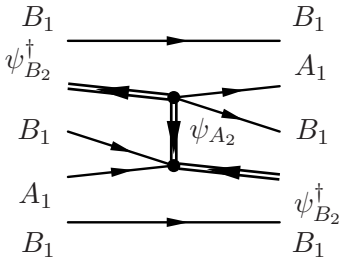
$$\Delta - J = D = \sqrt{\frac{1}{4} + 4 f(\lambda)^2 \sin^2(\pi p)} \tag{3.12}$$

By comparison with the two-loop spin chain Hamiltonian (3.4), we determine that $f(\lambda) \simeq \lambda$ in the weak 't Hooft coupling limit. Let us check this relation for the fermionic impurity

[1] Choosing the "vacuum" (3.7), one considers the breaking $\mathrm{SU}(4)_R \to \mathrm{SU}(2)_{G'} \times \mathrm{SU}(2)_G \times \mathrm{U}(1)$, and the vacuum preserves $\mathrm{SU}(2)_G \times \mathrm{U}(1)$. The extra U(1), which assigns charge $+1$ to $A_1, B_1^\dagger$ and charge -1 to $A_2, B_2^\dagger$, commutes with the generators of SU(2|2).

JHEP04(2009)066

$\psi^\dagger_{B_2}$. There is in fact only one diagram allowed by the index structure that contributes to the exchange of A_1 with $\psi^\dagger_{B_2}$ across a B_1 along the chain, as follows.



There is a factor of $16\pi^2\lambda^2$ coming from the F-term vertices, and a factor $1/2$ to convert to the anomalous dimension. The fermion propagator in position space is $i\not{x}/(4\pi|x|^3)$. The loop integral involved is

$$-i\int\frac{d^3yd^3z}{(4\pi)^7}\frac{(\not{x}-\not{z})(\not{z}-\not{y})\not{y}}{z^2(x-y)^2|y|^3|x-z|^3|y-z|^3} \tag{3.13}$$

whose logarithmically divergent part is

$$\begin{aligned}-2i\frac{\not{x}}{|x|^5}\int\frac{d^3yd^3z}{(4\pi)^7}\frac{(z-y)\cdot y}{z^2|y|^3|y-z|^3} &= -2i\frac{\not{x}}{|x|^5}\int\frac{d^3y}{(4\pi)^7|y|^3}y^\mu\frac{\partial}{\partial y^\mu}\int d^3z\frac{1}{z^2|y-z|}\\ &= \frac{1}{8\pi^2}\frac{i\not{x}\ln\Lambda}{(4\pi)^3|x|^5}\end{aligned} \tag{3.14}$$

The resulting anomalous dimension is identical to that of the A_2 (or $B_2^\dagger$) impurity, which is expected since they are in the same short multiplet.

3.3 Scattering and bound states

3.3.1 $(2|2)_A \otimes (2|2)_A$ sector

Now let us consider the scattering of a pair of basic $(2|2)$ impurities, working perturbatively at two-loop. First consider a pair of impurities both in the $(2|2)_A$ multiplet (or similarly, both in the $(2|2)_B$ multiplet), consisting of the fields $(A_2, B_2^\dagger; \psi^\dagger_{B_2})$. In particular, two A_2 impurities with momenta p_1 and p_2 scatter according to the Hamiltonian (3.4), and can form a bound state with dispersion relation [25]

$$\Delta - J - 1 = 2\lambda^2\sin^2(\pi p). \tag{3.15}$$

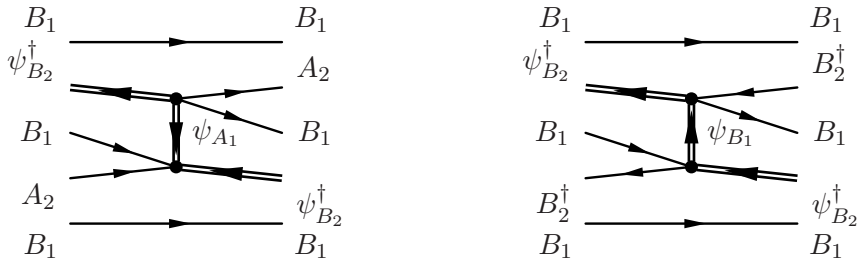
This saturates the BPS bound for the $(4|4)$ short multiplet of spin content $\{[2,0],[0,0]|[1,1]\}$ under $\mathrm{SU}(2)_G \times \mathrm{SU}(2)_r \subset \mathrm{SU}(2|2)$. The bosonic part of this short multiplet consists of the bound states of the pairs

$$A_2A_2,\ B_2^\dagger B_2^\dagger,\ A_2B_2^\dagger,\ \epsilon^{\alpha\beta}\left(\psi^\dagger_{B_2}\right)_\alpha\left(\psi^\dagger_{B_2}\right)_\beta \tag{3.16}$$

moving with momentum p. The wave function decays exponentially as the pair is separated along the chain. There is another $(4|4)$ "multiplet" of asymptotic scattering states of two $(2|2)_A$ multiplets, of spin content $\{[0,2],[0,0]|[1,1]\}$, whose bosonic part consists of

$$|A_2(p_1)B_2(p_2)^\dagger - B_2(p_1)^\dagger A_2(p_2)^\dagger\rangle,\quad \sigma^{\alpha\beta}_\mu|\left(\psi^\dagger_{B_2}\right)_\alpha(p_1)\left(\psi^\dagger_{B_2}\right)_\beta(p_2)\rangle \tag{3.17}$$

However, they cannot form bound states at two loop. This is easiest to see from the scattering of a bosonic $(2|2)_A$ impurity, say A_2, with a fermionic $(2|2)_A$ impurity $\psi^\dagger_{B_2}$. There is an exchange amplitude between A_2 and $\psi^\dagger_{B_2}$ (or B_2 and $\psi^\dagger_{B_2}$), as in the diagrams below, which allows only one bound state between them of given total momentum p. This bound state is already included in the $\{[2,0],[0,0]|[1,1]\}$ multiplet, and hence there are no fermionic bound states to pair up with potential bound states coming from (3.17).



It is plausible that the $(4|4)$ bound state of a pair of $(2|2)_A$ impurities, which we denote by $(4|4)_A$, remains a short multiplet at strong coupling.

3.3.2 $(2|2)_A \otimes (2|2)_B$ sector

From the earlier discussion on $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ sector of the spin chain we know that the A_2 and B_2 impurities from $(2|2)_A$ and $(2|2)_B$ do not interact at two-loop (but this is not necessarily the case for other pairs of impurities in $(2|2)_A \otimes (2|2)_B$). In particular we have two-impurity states, with A_2 of momentum p_1 and B_2 of momentum p_2, denoted by $|A_2(p_1)B_2(p_2)\rangle$, which are eigenstates of the two-loop dilatation operator. By SU(2|2) symmetry, there must be a 16-dimensional long multiplet of threshold (non-)scattering states, of spin content $\{[2,0],[0,2],[0,0],[0,0]|[1,1],[1,1]\}$. The $[2,0]$ part consists of the scalar triplet

$$|A_2(p_1)B_2(p_2)\rangle, \quad |B_2^\dagger(p_1)A_2^\dagger(p_2)\rangle, \quad |A_2(p_1)A_2^\dagger(p_2) - B_2^\dagger(p_1)B_2(p_2)\rangle. \tag{3.18}$$

The product representation $(2|2)_A \otimes (2|2)_B$ also consists of the $\mathrm{SU}(2)_r$ triplet of fermion bilinear

$$\sigma_\mu^{(\alpha\beta)}|(\psi_{B_2})^\dagger_\alpha(p_1)(\psi_{A_2})^\dagger_\beta(p_2)\rangle$$

Naively one may expect this to be the $[0,2]$ part of the long multiplet. However, these pairs of basic impurities are interacting at two-loop, and the corresponding states are not eigenstates of the Hamiltonian. In particular, the exchange amplitude

$$\sigma_\mu^{(\alpha\beta)}|\cdots(\psi_{B_2})^\dagger_\alpha(\psi_{A_2})^\dagger_\beta A_1\cdots\rangle \to \sigma_\mu^{(\alpha\beta)}|\cdots A_1(\psi_{A_2})^\dagger_\beta(\psi_{B_2})^\dagger_\alpha\cdots\rangle$$

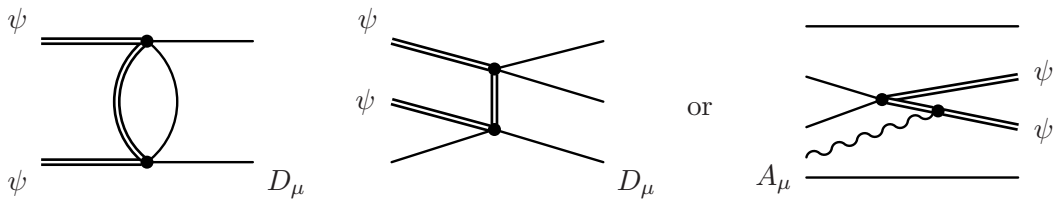
vanishes at two-loop, as the above diagram vanishes when the spinor indices α,β are symmetrized. This effect leads to a repulsive contact (i.e. nearest neighbor) interaction between the impurities $(\psi_{B_2})^\dagger_\alpha(p_1)$ and $(\psi_{A_2})^\dagger_\beta(p_2)$ in the $\mathrm{SU}(2)_r$ triplet sector.

The resolution to this seeming puzzle is due to operator mixing, between say $\sigma_\mu^{(\alpha\beta)}|(\psi_{B_2})^\dagger_\alpha(p_1)(\psi_{A_2})^\dagger_\beta(p_2)\rangle$ and $|D_\mu(p)\rangle$, the state of an impurity $D_\mu A_1$ or $D_\mu B_1$ moving at momentum $p = p_1 + p_2$. At two-loop this can be computed from the amplitude

$$\sigma_\mu^{(\alpha\beta)}|\cdots(\psi_{B_2})^\dagger_\alpha(\psi_{A_2})^\dagger_\beta\cdots\rangle \to |\cdots A_1 D_\mu B_1\cdots\rangle$$

JHEP04(2009)066

via the following Feynman diagrams



It will turn out that we can determine the coefficients of these amplitudes simply based on the consistency requirement that there are threshold non-scattering states of such mixed operators.

A simple example of such mixing at zero momentum (more precisely, at momentum $p = 1$) is the following protected operator obtained by acting on the vacuum chain with supercharges,

$$\begin{aligned}
(Q_{13})_\alpha (Q_{24})_\beta |A_1 B_1 A_1 B_1 \cdots\rangle = &- \sum_{n=even,\, m=odd} |(\psi_{B_2})^\dagger_\beta(n)(\psi_{A_2})^\dagger_\alpha(m)\rangle \\
&+ i\sigma^\mu_{\alpha\beta} \sum_{n\ even} |D_\mu A_1(n)\rangle - i\epsilon_{\alpha\beta} \\
&\times \left(\sum_{n\ odd} |\sigma_{24}(n)\rangle - \sum_{n\ even} |\tilde\sigma_{24}(n)\rangle \right) \\
(Q_{24})_\beta (Q_{13})_\alpha |A_1 B_1 A_1 B_1 \cdots\rangle = &\sum_{n=even,\, m=odd} |(\psi_{B_2})^\dagger_\beta(n)(\psi_{A_2})^\dagger_\alpha(m)\rangle \\
&+ i\sigma^\mu_{\alpha\beta} \sum_{n\ odd} |D_\mu B_1(n)\rangle + i\epsilon_{\alpha\beta} \\
&\times \left(\sum_{n\ odd} |\sigma_{24}(n)\rangle - \sum_{n\ even} |\tilde\sigma_{24}(n)\rangle \right)
\end{aligned} \tag{3.19}$$

where σ_{24} and $\tilde\sigma_{24}$ are defined as

$$\begin{aligned}
\sigma_{24} &= \frac{2\pi}{k}\left(-\bar\phi^1\phi_1 + \bar\phi^2\phi_2 - \bar\phi^3\phi_3 + \bar\phi^4\phi_4\right), \\
\tilde\sigma_{24} &= \frac{2\pi}{k}\left(-\phi_1\bar\phi^1 + \phi_2\bar\phi^2 - \phi_3\bar\phi^3 + \phi_4\bar\phi^4\right).
\end{aligned} \tag{3.20}$$

The sum of the two lines in (3.19) gives the total derivative of the vacuum chain, whereas the difference gives another protected operator (in both the $SU(2)_r$ triplet and singlet sector). A special case is when the length of the chain is 2, and we obtain a component of the $SU(4)_R$ current

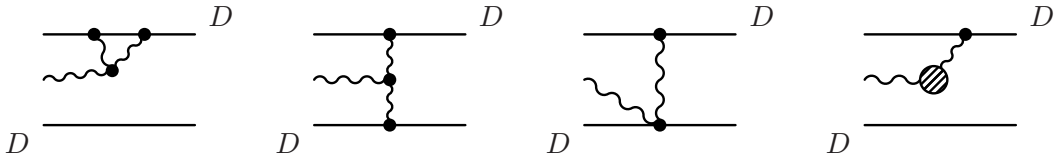
$$(J_\mu)_1{}^3 = \mathrm{Tr}\left[A_1 D_\mu B_1 - (D_\mu A_1)\, B_1 - i\,(\psi_{B_2})^\dagger_\alpha\,(p_1)\sigma^{\alpha\beta}_\mu\,(\psi_{A_2})^\dagger_\beta\,(p_2)\right] \tag{3.21}$$

Let us now compute the operator mixing in the sector of a $(\psi_{B_2})^\dagger_\alpha$ and a $(\psi_{A_2})^\dagger_\beta$ impurity in the triplet of $SU(2)_r$, or a single impurity D_μ. Denote by $|n, m\rangle$ the state with $(\psi_{B_2})^\dagger_\alpha$ at position $2n$ and $(\psi_{A_2})^\dagger_\beta$ at position $2m+1$, with the spinor indices contracted by

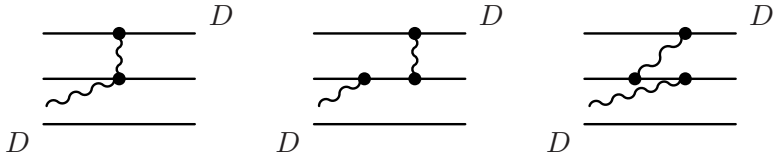
$\sigma_\mu^{\alpha\beta}$. Denote by $|D(n)\rangle$ the state of a D_μ acting on the site $2n$, and by $|D'(n)\rangle$ the state of D_μ acting on the site $2n+1$. The two-loop dilatation operator then acts on these states as

$$\begin{aligned}
H|n,m\rangle &= -\lambda^2(|n-1,m\rangle + |n+1,m\rangle + |n,m-1\rangle + |n,m+1\rangle - 4|n,m\rangle), \\
&\qquad\qquad \left(|n-m-\frac{1}{2}| > \frac{1}{2}\right) \\
H|n,n\rangle &= -\lambda^2(|n-1,n\rangle + |n,n+1\rangle - (4-\delta)|n,n\rangle) \\
&\quad - c_1\lambda^2(|D(n)\rangle - |D'(n)\rangle) - c_2\lambda^2(|D'(n-1)\rangle - |D(n+1)\rangle), \\
H|n,n-1\rangle &= -\lambda^2(|n+1,n-1\rangle + |n,n-2\rangle - (4-\delta)|n,n-1\rangle) \\
&\quad - c_1\lambda^2(|D'(n-1)\rangle - |D(n)\rangle) - c_2\lambda^2(|D(n-1)\rangle - |D'(n)\rangle), \\
H|D(n)\rangle &= -c_1\lambda^2(|n,n\rangle - |n,n-1\rangle) - c_2\lambda^2(|n+1,n\rangle - |n-1,n-1\rangle) \\
&\quad - c_3\lambda^2(|D'(n-1)\rangle + |D'(n)\rangle - 2|D(n)\rangle), \\
&\quad - c_4\lambda^2(|D(n-1)\rangle + |D(n+1)\rangle - 2|D(n)\rangle), \\
H|D'(n)\rangle &= -c_1\lambda^2(|n+1,n\rangle - |n,n\rangle) - c_2\lambda^2(|n+1,n+1\rangle - |n,n-1\rangle) \\
&\quad - c_3\lambda^2(|D(n)\rangle + |D(n+1)\rangle - 2|D'(n)\rangle) \\
&\quad - c_4\lambda^2(|D'(n-1)\rangle + |D'(n+1)\rangle - 2|D'(n)\rangle).
\end{aligned} \tag{3.22}$$

where $\delta, c_1, c_2, c_3, c_4$ are constants that can be computed from the two-loop diagrams. In particular, δ is a potentially nonzero correction to the anomalous dimension when $(\psi_{B_2})^\dagger_\alpha$ and $(\psi_{A_2})^\dagger_\beta$ are next to each other (although it will turn out to be zero in this case). The coefficients c_1 and c_2 are due to the mixing between adjacent fermion pair $(\psi_{B_2})^\dagger_\alpha(\psi_{A_2})^\dagger_\beta$ and a D_μ on the nearest and next-to-nearest neigboring sites, respectively. c_3 is due to the mixing of a D_μ impurity with another D_μ on nearest neighboring sites, according to the diagrams



whereas c_4 is due to mixing of D_μ's on next-to-nearest neighboring sites, from the following diagrams



We will not compute these diagrams directly, but simply determine them from the existence of the threshold states at general momenta below. The result is

$$\delta = 0, \quad c_1 = \frac{1}{\sqrt{2}}, \quad c_2 = -\frac{1}{\sqrt{2}}, \quad c_3 = 1, \quad c_4 = \frac{1}{2}. \tag{3.23}$$

A general state of total momentum p takes the form

$$|\Psi\rangle = \sum_{n,m} e^{\pi i p(n+m+\frac{1}{2})} f(n-m)|n,m\rangle + g\sum_n e^{2\pi i p n}|D(n)\rangle + g'\sum_n e^{2\pi i p(n+\frac{1}{2})}|D'(n)\rangle \tag{3.24}$$

Suppose $|\Psi\rangle$ is an energy eigenstate $H|\Psi\rangle = \lambda^2 E|\Psi\rangle$. This is equivalent to the equations

$$\begin{aligned}
&2\cos(\pi p)(f(n-1)+f(n+1)) - 4f(n) = -Ef(n), \qquad n \geq 2 \text{ or } n \leq -1\\
&2\cos(\pi p)f(2) - 4f(1) + \sqrt{2}\cos(\pi p)(-e^{-\pi i p/2}g + e^{\pi i p/2}g') = -Ef(1),\\
&2\cos(\pi p)f(-1) - 4f(0) + \sqrt{2}\cos(\pi p)(-e^{-\pi i p/2}g' + e^{\pi i p/2}g) = -Ef(0),\\
&(\cos(2\pi p)-1)g + 2(\cos(\pi p)g' - g) + \sqrt{2}\cos(\pi p)(-e^{\pi i p/2}f(1) + e^{-\pi i p/2}f(0)) = -Eg,\\
&(\cos(2\pi p)-1)g' + 2(\cos(\pi p)g' - g) + \sqrt{2}\cos(\pi p)(-e^{\pi i p/2}f(0) + e^{-\pi i p/2}f(1)) = -Eg',
\end{aligned} \tag{3.25}$$

The threshold state is given by

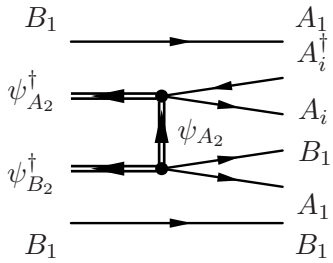
$$\begin{aligned}
f(n) &= -\frac{i}{\sqrt{2}}e^{-i(n-1/2)\alpha}\left(\cos(\pi p) - e^{-i\alpha}\right), \qquad (n \geq 1)\\
f(n) &= -\frac{i}{\sqrt{2}}e^{-i(n-1/2)\alpha}\left(\cos(\pi p) - e^{i\alpha}\right), \qquad (n \leq 0)\\
g &= \sin\left(\frac{\pi p - \alpha}{2}\right), \qquad g' = \sin\left(\frac{\pi p + \alpha}{2}\right),\\
E &= 4\sin^2\left(\frac{\pi p - \alpha}{2}\right) + 4\sin^2\left(\frac{\pi p + \alpha}{2}\right).
\end{aligned} \tag{3.26}$$

where $\alpha = \pi(p_1 - p_2)$ is the difference between the momenta of the two ψ impurities. In particular the protected operators in the triplet sector of (3.19) are given by the special case $p = 1$, $\alpha = 0$. One can also check that there are no bound states at two loop.[2] A priori these threshold states may not survive at higher loops, but they may survive in the pp-wave limit as unbound $(2|2)_A$ and $(2|2)_B$ impurities.

There is also operator mixing in the $[0,0]$ part of the long multiplet. For instance, the fermion bilinear singlet

$$|\psi^\dagger_{B_2}(p_1)\psi^\dagger_{A_2}(p_2)\rangle = \epsilon^{\alpha\beta}|(\psi_{B_2})^\dagger_\alpha(p_1)(\psi_{A_2})^\dagger_\beta(p_2)\rangle \tag{3.27}$$

can mix with four bosons, via diagrams such as the following



We have seen this mixing at zero momentum already in (3.19).

[2] An attempt to find such bound states is to set say $e^{i\alpha} = \cos(\pi p)$ with purely imaginary α in (3.25), but this state is growing as opposed to decaying, exponentially, in the separation between $\psi^\dagger_{B_2}$ and $\psi^\dagger_{A_2}$.

4 Penrose limit of type IIA string theory on $AdS_4 \times \mathbb{CP}^3$

The 't Hooft limit of the $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory is dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$ [1]. The metric on $AdS_4 \times \mathbb{CP}^3$ can be written as [27]

$$\begin{aligned} ds^2 = R^2 \Big\{ & -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_2^2 \\ & + 4d\mu^2 + 4\sin^2\mu \Big[d\alpha^2 + \frac{1}{4}\sin^2\alpha \left(\sigma_1^2 + \sigma_2^2 + \cos^2\alpha\sigma_3^2\right) \\ & + \frac{1}{4}\cos^2\mu \left(d\chi + \sin^2\alpha\sigma_3\right)^2 \Big] \Big\} \end{aligned} \tag{4.1}$$

Here R is the radius of the AdS_4, and $\sigma_{1,2,3}$ are left-invariant 1-forms on an S^3, parameterized by (θ, ϕ, ψ),

$$\begin{aligned} \sigma_1 &= \cos\psi d\theta + \sin\psi\sin\theta d\phi, \\ \sigma_2 &= \sin\psi d\theta - \cos\psi\sin\theta d\phi, \\ \sigma_3 &= d\psi + \cos\theta d\phi. \end{aligned} \tag{4.2}$$

The range of the coordinates is $0 \le \mu, \alpha \le \pi/4\,, 0 \le \theta \le \pi\,, 0 \le \phi \le 2\pi\,, 0 \le \chi, \psi \le 4\pi$. The Penrose limit is defined by focusing on the geodesic along χ, with $\mu = \pi/4$, $\alpha = 0$, $\rho = 0$. To do this we can define the new variables

$$\rho = \frac{\tilde{\rho}}{R}, \quad \mu = \frac{\pi}{4} + \frac{u}{2R}, \quad \alpha = \frac{r}{\sqrt{2}R}, \quad dx^+ = \frac{dt + d\chi/2}{2}, \quad dx^- = R^2\frac{dt - d\chi/2}{2}, \tag{4.3}$$

and scale $R \to \infty$. The metric then reduces to

$$\begin{aligned} ds^2 &= -4dx^+dx^- + du^2 + d\tilde{\rho}^2 + \tilde{\rho}^2 d\Omega_2^2 + dr^2 + \frac{r^2}{4}\sum_{i=1}^{3}\sigma_i^2 - (u^2 + \tilde{\rho}^2)(dx^+)^2 + \frac{1}{2}r^2\sigma_3 dx^+ \\ &= -4dx^+dx^- + du^2 + \sum_{i=1}^{3} dy_i^2 + \sum_{j=1}^{2} dz_j d\bar{z}_j - (u^2 + \sum_{i=1}^{3} y_i^2)(dx^+)^2 \\ &\quad - \frac{i}{2}\sum_{j=1}^{2}(\bar{z}_j dz_j - z_j d\bar{z}_j)dx^+ \end{aligned} \tag{4.4}$$

where z_1, z_2 are standard complex coordintes on the $\mathbb{C}^2$ with radial coordinates (r, θ, ϕ, ψ). To put the metric in standard pp-wave form, we make a further coordinate change

$$z_j = e^{-ix^+/2}w_j, \qquad \bar{z}_j = e^{ix^+/2}\bar{w}_j, \tag{4.5}$$

and the metric becomes

$$ds^2 = -4dx^+dx^- + du^2 + \sum_{i=1}^{3} dy_i^2 + \sum_{j=1}^{2} dw_j d\bar{w}_j - \left(u^2 + \sum_{i=1}^{3} y_i^2 + \frac{1}{4}\sum_{j=1}^{2}|w_j|^2\right)\left(dx^+\right)^2 \tag{4.6}$$

There are also fluxes in the $AdS_4 \times \mathbb{C}P^3$ background, reducing to

$$\begin{aligned} F_2 &= -dx^+ \wedge du, \\ F_4 &= -3dx^+ \wedge dy_1 \wedge dy_2 \wedge dy_3, \end{aligned} \tag{4.7}$$

in the Penrose limit. This pp-wave solution was found in [28] (see also [29]), and preserves 24 supersymmetries as the $AdS_4 \times \mathbb{C}P^3$ background does. We shall organize the coordinates (u, y_i) as (X^1, X^2, X^3, X^4), and $w_i, \bar{w}_i$ as (X^5, X^6, X^7, X^8). In the light cone gauge $X^+ = \tau$, $\Gamma^+\Theta = 0$, the Green-Schwarz action for the type IIA string is (we follow the conventions of [30]; for earlier studies of the GS string in this pp-wave background see [31, 32])

$$\begin{aligned} S = \frac{1}{2\pi\alpha'} \int dt \int_0^{2\pi\alpha' p^+} d\sigma \Bigg\{ & \frac{1}{2} \sum_{i=1}^{8} \left[(\dot{X}^i)^2 - (X^{i'})^2 \right] - \frac{1}{2} \sum_{i=1}^{4} (X^i)^2 - \frac{1}{8} \sum_{j=5}^{8} (X_j)^2 \\ & - i\bar{\Theta}\Gamma^- \left[\partial_\tau + \Gamma^{11}\partial_\sigma - \frac{1}{4}\Gamma^1\Gamma^{11} - \frac{3}{4}\Gamma^{234} \right] \Theta \Bigg\} \end{aligned} \tag{4.8}$$

The bosonic excitations of the type IIA string in this pp-wave background have light cone spectrum

$$H = \sum_{i=1}^{4} \sum_{n=-\infty}^{\infty} N_n^{(i)} \sqrt{1 + \frac{n^2}{(\alpha' p^+)^2}} + \sum_{j=5}^{8} \sum_{n=-\infty}^{\infty} N_n^{(j)} \sqrt{\frac{1}{4} + \frac{n^2}{(\alpha' p^+)^2}} \tag{4.9}$$

In terms of the gauge theory spin chain variables, $p^+ = J/R^2$,[3] $p = n/J$, $R^2/\alpha' = \pi\sqrt{2\lambda}$ [1], we find the dispersion relations

$$\begin{aligned} E^{(i)} &= \sqrt{1 + 2\lambda(\pi p)^2}, \qquad i = 1, \cdots, 4, \\ E^{(j)} &= \sqrt{\frac{1}{4} + 2\lambda(\pi p)^2}, \qquad j = 5, \cdots, 8. \end{aligned} \tag{4.10}$$

It follows from the fermion equation of motion that

$$\begin{aligned} (\partial_\tau^2 - \partial_\sigma^2)\Theta &= \left(\partial_\tau - \Gamma^{11}\partial_\sigma\right)\left(\partial_\tau + \Gamma^{11}\partial_\sigma\right)\Theta \\ &= -\left(\frac{1}{4}\Gamma^1\Gamma^{11} + \frac{3}{4}\Gamma^{234}\right)^2 \Theta \\ &= \left(\frac{5}{8} + \frac{3}{8}\Gamma^{1234}\Gamma^{11}\right)\Theta \end{aligned} \tag{4.11}$$

Hence there are four fermions of mass 1, satisfying $\Gamma^{1234}\Gamma^{11}\Theta = \Gamma^{5678}\Theta = \Theta$, and four fermions of mass 1/2, satisfying $\Gamma^{1234}\Gamma^{11}\Theta = \Gamma^{5678}\Theta = -\Theta$. Consequently the fermion spectrum takes the same form as the bosonic one (4.9). Note that the Green-Schwarz action has symmetry group $\mathrm{SU}(2)' \times \mathrm{SU}(2|2)$, which contains bosonic subgroup $\mathrm{SU}(2)' \times \mathrm{SU}(2)_G \times \mathrm{SU}(2)_r \times \mathrm{U}(1)_D$. Here $\mathrm{SU}(2)' \times \mathrm{SU}(2)_G \simeq \mathrm{SO}(4)$ is the rotation group on (X^5, X^6, X^7, X^8),

[3] To see this, note that $p^+ = -\frac{1}{2}p_- = \frac{i}{2R^2}\frac{\partial}{\partial x^-} = \frac{i}{R^2}(\frac{1}{2}\partial_t - \partial_\chi)$. Since $\chi \sim \chi + 4\pi$, the charge quantization is such that $-i\partial_\chi = J/2$, and $i\partial_t = \Delta$. For the chiral primary with $\Delta = J$ (J is the length of the alternating $A_1 B_1$ chain divided by 2), we have $p^+ = J/R^2$.

whereas $\mathrm{SU}(2)_r$ rotates (X^2, X^3, X^4). The supersymmetry transformations of the X^i's take the form

$$\begin{aligned} \delta_{A\alpha} u &\sim \Theta_{A\alpha}, \\ \delta_{A\alpha} y^i &\sim (\sigma^i)_\alpha{}^\beta \Theta_{A\beta}, \\ \delta_{A\alpha} X^{B\dot{C}} &\sim \delta_A^B \Theta_\alpha^{\dot{C}}, \end{aligned} \tag{4.12}$$

where $X^{A\dot{B}}$ stand for (X^5, X^6, X^7, X^8) in $\mathrm{SU}(2)_G \times \mathrm{SU}(2)'$ bispinor notation. This is consistent with the fact that $\Theta_{A\alpha}$ (satisfying $\Gamma^{5678}\Theta = \Theta$) have the same mass as (u, y^i), and $\Theta_{\dot{A}\alpha}$ (satisfying $\Gamma^{5678}\Theta = -\Theta$) have the same mass as $X^{A\dot{B}}$. The $\mathrm{SU}(2)'$ symmetry appears to be an accidental symmetry in the pp-wave limit, and reduces to a U(1) away from the Penrose limit.

We shall note an important difference of this pp-wave limit from say the BMN scaling of $\mathcal{N} = 4$ SYM [26]: the limit here is defined by taking $\lambda, J \to \infty$, while keeping λ/J^2 fixed. This may appear surprising from perturbative gauge theory, as we might have expected from the two-loop dispersion relation (3.5) that the BMN scaling might be defined by λ/J kept fixed. On other hand, in general the ℓ-loop corrections may contribute to the dispersion relation in the form

$$E^{(\ell)} = \lambda^\ell \sum_{n=1}^{\lfloor \ell/2 \rfloor} c_{\ell,n} \sin^{2n}(\pi p) \tag{4.13}$$

where $c_{\ell,n}$ are generically nonzero (say for $n = 1$ and $\ell > 2$), and hence alters the form of the BMN scaling at strong coupling. This indeed seems to happen in $\mathcal{N} = 6$ CSM theory.

At classical dimension 1/2, there are 4 bosonic and 4 fermionic excitations. They are the modes of $(X^{A\dot{B}}, \Theta_\alpha^{\dot{B}})$, where $\dot{B}$ is an $\mathrm{SU}(2)'$ spinor index, and lie in the two short multiplets $(2|2)_A$ and $(2|2)_B$ with respect to SU(2|2). Their exact dispersion relation at general 't Hooft coupling is given by the BPS bound (3.12), where the function $f(\lambda)$ scales differently with λ in the weak and strong coupling limits, see (3.5) and (4.10)

$$\begin{aligned} f(\lambda) &\sim \lambda, \qquad \lambda \ll 1, \\ f(\lambda) &\sim \sqrt{\lambda/2}, \qquad \lambda \gg 1. \end{aligned} \tag{4.14}$$

A similar phenomenon was observed in [29]. This is in contrast with $\mathcal{N} = 4$ SYM, where the central charge of the extended superconformal algebra of the infinite spin chain is proportional to $\sqrt{\lambda}$ in both the weak and strong coupling limits (although there is no reason why this should be true at general finite 't Hooft coupling, as pointed out in [33]).

At classical dimension 1, we have pairs of free excitations in $(2|2)_A \otimes (2|2)_A$, $(2|2)_B \otimes (2|2)_B$, $(2|2)_A \otimes (2|2)_B$, as well as an additional $(4|4)$ multiplet of spin content $\{[0,2],[0,0]|[1,1]\}$ under $\mathrm{SU}(2)_G \times \mathrm{SU}(2)_r$, which are the modes of $(y^i, u, \Theta_{A\alpha})$. Note that the dispersion relation of the $(4|4)$ multiplet is consistent with the form of the BPS bound for $(4|4)$ short multiplets at generic coupling,

$$D = \sqrt{1 + 4 f(\lambda)^2 \sin^2(\pi p)} \tag{4.15}$$

It is plausible that this (4|4) multiplet survives as a short multiplet away from the pp-wave limit. Naively, we may expect this multiplet to include the D_μ impurities. But as we have seen at two-loop, the D_μ impurities mix with the $(2|2)_A \otimes (2|2)_B$ sector to form threshold scattering states, and there are no bound states (at least not at two loop). It is a puzzle to us how to describe this (4|4) multiplet perturbatively in the gauge theory, if it exists.

In the $(2|2)_A \otimes (2|2)_A$ (or $(2|2)_B \otimes (2|2)_B$) sector, at two-loop we have found bound states that saturate the BPS bound; they may survive as short multiplets at finite coupling, and may become free pairs of $(2|2)_A$ (or $(2|2)_B$) excitations in the pp-wave limit. In $(2|2)_A \otimes (2|2)_B$ sector, we have found an $8+8$ long multiplet of threshold (non-)scattering states at two-loop. It is unclear whether these survive at finite coupling, and match onto the free pairs of $(2|2)_A$ and $(2|2)_B$ excitations in the pp-wave limit.

5 Giant magnons

It is easy to find giant magnon solutions to the Nambu action in $AdS_4 \times \mathbb{CP}^3$, following [33]. Corresponding to our vacuum spin chain is a string moving along a geodesic in the $\mathbb{CP}^3$, with $\mu = \pi/4$ and $\alpha = 0$, parameterized by χ, in the coordinate system of (4.1). Alternatively, we can work with projective coordinates $[z_1, z_2, z_3, z_4]$, and consider the geodesic given by $|z_1| = |z_3|$, $z_2 = z_4 = 0$. The first type of giant magnons move on the S^2 parameterized by χ and μ, at $\alpha = 0$. In projective coordinates, this is the $\mathbb{CP}^1$ defined by $z_2 = z_4 = 0$. Note that this sphere preserves the $\mathrm{SU}(2)_G$ which rotates z_2 and z_4. In particular it is consistent to restrict the giant magnon solution to this S^2. The S^2, or $\mathbb{CP}^1$, has its radius equal to the AdS_4 radius $R = \sqrt{\pi\alpha'}(2\lambda)^{\frac{1}{4}}$. The giant magnon solution takes the identical form as the one in [33], with dispersion relation

$$E - J = \sqrt{2\lambda}|\sin\left(\frac{\Delta\chi}{4}\right)| = \sqrt{2\lambda}|\sin(\pi p)| \tag{5.1}$$

where the angular difference between two ends of the magnon, $\Delta\chi/2$, is identified with $2\pi p$ (χ has periodicity 4π). This is consistent with (and saturates) the large λ limit of the BPS bound due to the centrally extended SU(2|2) algebra of the infinite chain.

Interestingly, there is a second class of giant magnons, which lie in an $\mathbb{RP}^2 \subset \mathbb{CP}^3$, rather than the $\mathbb{CP}^1$. The $\mathbb{RP}^2$'s that contain the geodesic $|z_1| = |z_3|$, $z_2 = z_4 = 0$ are defined by

$$|z_1| = |z_3|, \qquad \frac{z_2}{z_1 - z_3} = \alpha x, \qquad \frac{z_4}{z_1 - z_3} = \beta x, \qquad x \in \mathbb{R}. \tag{5.2}$$

This is seen more explicitly in rotated coordinates $(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4) = (\frac{z_1 - z_3}{\sqrt{2}}, z_2, i\frac{z_1 + z_3}{\sqrt{2}}, z_4)$, where the original geodesic is $\tilde{z}_3/\tilde{z}_1 \in \mathbb{R}$, $\tilde{z}_2 = \tilde{z}_4 = 0$, and the defining equation of the $\mathbb{RP}^2$ becomes

$$\frac{\tilde{z}_3}{\tilde{z}_1} \in \mathbb{R}, \qquad \frac{\tilde{z}_2}{\tilde{z}_1} = \alpha x, \qquad \frac{\tilde{z}_4}{\tilde{z}_1} = \beta x, \qquad x \in \mathbb{R}. \tag{5.3}$$

where α and β are complex constants, and we have a family of $\mathbb{RP}^2$'s parameterized by (α, β), which transform as a doublet under $\mathrm{SU}(2)_G$. In particular, a given $\mathbb{RP}^2$ lies in a $\mathbb{CP}^2 \subset \mathbb{CP}^3$ which is fixed by a U(1) symmetry, and it is the fixed locus of an involution of the $\mathbb{CP}^2$. It is therefore consistent to restrict the giant magnon solution to this $\mathbb{RP}^2$. We

JHEP04(2009)066

can describe the $\mathbb{RP}^2$ as the quotient of an auxiliary sphere $\tilde{S}^2$ by the antipodal map. Note however that this $\tilde{S}^2$ has radius $2R$. Hence the giant magnon solutions that move along the geodesic, which is half the equator of $\tilde{S}^2$ (with ends identified by the antipodal map), has dispersion relation

$$E - J = 2\sqrt{2\lambda}|\sin\left(\frac{\Delta\varphi}{2}\right)| \tag{5.4}$$

where φ is the angular variable on the equator of $\tilde{S}^2$, ranging from 0 to 2π, and $\Delta\varphi$ is the difference between the two ends of the giant magnon. On the $\mathbb{RP}^2$, however, φ is identified with periodicity π, and it is natural to propose the identification with spin chain momentum $\Delta\varphi = \pi p$. So we obtain the dispersion relation

$$E - J = 2\sqrt{2\lambda}|\sin(\pi p/2)| \tag{5.5}$$

Note that with given $0 < p < 1$, there is another giant magnon with $\Delta\varphi = \pi(1-p)$ with the same ends as the one with $\Delta\varphi = \pi p$, and has dispersion relation $E - J = 2\sqrt{2\lambda}|\cos(\pi p/2)|$. The minimal energy configuration carrying momentum p should then be

$$E - J = 2\sqrt{2\lambda}\min\{|\sin(\pi p/2)|, |\cos(\pi p/2)|\} \tag{5.6}$$

Note that this obeys the large λ limit of the BPS bound (3.12), but does not saturate it.

Naively, based on the transformation under $SU(2)_G$, one may want to identify the first type of giant magnons with the (4|4) multiplet in the pp-wave limit, since it involves excitations in the u-direction (see (4.6)), and to identify the second type of giant magnons with the (2|2) multiplets. However, the second type of giant magnons does not saturate the BPS bound of the SU(2|2) algebra, and should correspond to long multiplets. A potential resolution to this puzzle is that there are fermion zero modes of the giant magnons, which carry additional representations of the $SU(2)_G$. It is therefore tnot clear to us how to identify these giant magnons with the excitations in the pp-wave limit or in perturbative gauge theory.

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References

[1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, $N = 6$ *superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

JHEP04(2009)066

[2] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D 75** (2007) 045020 [hep-th/0611108] [SPIRES]; *Gauge symmetry and supersymmetry of multiple M2-branes*, *Phys. Rev.* **D 77** (2008) 065008 [arXiv:0711.0955] [SPIRES]; *Comments on multiple M2-branes*, *JHEP* **02** (2008) 105 [arXiv:0712.3738] [SPIRES].

[3] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys.* **B 811** (2009) 66 [arXiv:0709.1260] [SPIRES].

[4] M. Van Raamsdonk, *Comments on the Bagger-Lambert theory and multiple M2-branes*, *JHEP* **05** (2008) 105 [arXiv:0803.3803] [SPIRES].

[5] J. Distler, S. Mukhi, C. Papageorgakis and M. Van Raamsdonk, *M2-branes on M-folds*, *JHEP* **05** (2008) 038 [arXiv:0804.1256] [SPIRES].

[6] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, *Supersymmetric Yang-Mills theory from Lorentzian three-algebras*, *JHEP* **08** (2008) 094 [arXiv:0806.0738] [SPIRES].

[7] P.-M. Ho, Y. Imamura and Y. Matsuo, *M2 to D2 revisited*, *JHEP* **07** (2008) 003 [arXiv:0805.1202] [SPIRES].

[8] A. Gustavsson, *One-loop corrections to Bagger-Lambert theory*, *Nucl. Phys.* **B 807** (2009) 315 [arXiv:0805.4443] [SPIRES].

[9] M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence*, *JHEP* **09** (2008) 072 [arXiv:0806.1519] [SPIRES].

[10] D. Gaiotto and X. Yin, *Notes on superconformal Chern-Simons-matter theories*, *JHEP* **08** (2007) 056 [arXiv:0704.3740] [SPIRES].

[11] J.H. Schwarz, *Superconformal Chern-Simons theories*, *JHEP* **11** (2004) 078 [hep-th/0411077] [SPIRES].

[12] A.N. Kapustin and P.I. Pronin, *Nonrenormalization theorem for gauge coupling in $(2+1)$-dimensions*, *Mod. Phys. Lett.* **A 9** (1994) 1925 [hep-th/9401053] [SPIRES].

[13] W. Chen, G.W. Semenoff and Y.-S. Wu, *Two loop analysis of nonAbelian Chern-Simons theory*, *Phys. Rev.* **D 46** (1992) 5521 [hep-th/9209005] [SPIRES].

[14] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, *Renormalizations in supersymmetric and nonsupersymmetric nonAbelian Chern-Simons field theories with matter*, *Nucl. Phys.* **B 391** (1993) 333 [SPIRES].

[15] L.V. Avdeev, G.V. Grigorev and D.I. Kazakov, *Renormalizations in abelian Chern-Simons field theories with matter*, *Nucl. Phys.* **B 382** (1992) 561 [SPIRES].

[16] J.A. Minahan and K. Zarembo, *The Bethe ansatz for superconformal Chern-Simons*, *JHEP* **09** (2008) 040 [arXiv:0806.3951] [SPIRES].

[17] Y. Imamura and K. Kimura, *On the moduli space of elliptic Maxwell-Chern-Simons theories*, *Prog. Theor. Phys.* **120** (2008) 509 [arXiv:0806.3727] [SPIRES].

[18] Y. Honma, S. Iso, Y. Sumitomo and S. Zhang, *Scaling limit of $N = 6$ superconformal Chern-Simons theories and Lorentzian Bagger-Lambert theories*, *Phys. Rev.* **D 78** (2008) 105011 [arXiv:0806.3498] [SPIRES].

[19] T. Nishioka and T. Takayanagi, *On type IIA penrose limit and $N = 6$ Chern-Simons theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391] [SPIRES].

JHEP04(2009)066

[20] J. Bhattacharya and S. Minwalla, *Superconformal indices for $\mathcal{N} = 6$ Chern Simons theories*, *JHEP* **01** (2009) 014 [arXiv:0806.3251] [SPIRES].

[21] N. Beisert, *The dilatation operator of $N = 4$ super Yang-Mills theory and integrability*, *Phys. Rept.* **405** (2005) 1 [hep-th/0407277] [SPIRES].

[22] N. Beisert, *The* SU(2|2) *dynamic S-matrix*, *Adv. Theor. Math. Phys.* **12** (2008) 945 [hep-th/0511082] [SPIRES].

[23] N. Beisert, *The analytic Bethe ansatz for a chain with centrally extended* SU(2|2) *symmetry*, *J. Stat. Mech.* (2007) P01017 [nlin/0610017] [SPIRES].

[24] J.A. Minahan and K. Zarembo, *The Bethe-ansatz for $N = 4$ super Yang-Mills*, *JHEP* **03** (2003) 013 [hep-th/0212208] [SPIRES].

[25] L.D. Faddeev, *How algebraic Bethe ansatz works for integrable model*, hep-th/9605187 [SPIRES].

[26] D.E. Berenstein, J.M. Maldacena and H.S. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021] [SPIRES].

[27] C.N. Pope and N.P. Warner, *An* SU(4) *invariant compactification of $D = 11$ supergravity on a stretched seven sphere*, *Phys. Lett.* **B 150** (1985) 352 [SPIRES].

[28] I. Bena and R. Roiban, *Supergravity pp-wave solutions with 28 and 24 supercharges*, *Phys. Rev.* **D 67** (2003) 125014 [hep-th/0206195] [SPIRES].

[29] H. Lin and J.M. Maldacena, *Fivebranes from gauge theory*, *Phys. Rev.* **D 74** (2006) 084014 [hep-th/0509235] [SPIRES].

[30] J. Michelson, *A pp-wave with 26 supercharges*, *Class. Quant. Grav.* **19** (2002) 5935 [hep-th/0206204] [SPIRES].

[31] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on pp-wave*, *Nucl. Phys.* **B 644** (2002) 128 [hep-th/0208029] [SPIRES].

[32] S.-j. Hyun and H.-j. Shin, *$N = (4, 4)$ type IIA string theory on pp-wave background*, *JHEP* **10** (2002) 070 [hep-th/0208074] [SPIRES].

[33] D.M. Hofman and J.M. Maldacena, *Giant magnons*, *J. Phys.* **A 39** (2006) 13095 [hep-th/0604135] [SPIRES].

JHEP04(2009)066